

Dirac gauginos and unification in F-theory

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Abstract

Supersymmetric models in which the gauginos acquire Dirac masses, rather than Majorana masses, offer an appealing alternative to the minimal supersymmetric standard model, especially in the light of the bounds set on superpartner masses by the 2011 LHC data. Dirac gauginos require the presence of chiral multiplets in the adjoint representation of the gauge group, and the realisation of such scenarios in F-theory is the subject of this paper. The chiral adjoints drastically alter the usual picture of gauge coupling unification, but this is disturbed anyway in F-theory models with non-trivial hypercharge flux. The interplay between these two factors is explored, and it is found for example that viable F-theory unification can be achieved at around the reduced Planck scale, if there is an extra vector-like pair of singlet leptons with TeV-scale mass. I then discuss the conditions which must be satisfied by the geometry and hypercharge flux of an F-theory model with Dirac gauginos. One nice possibility is for the visible sector to be localised on a $K3$ surface, and this is discussed in some detail. Finally, I describe how to achieve an unbroken discrete R -symmetry in such compactifications, which is an important ingredient in many models with Dirac gauginos, and write down a simple example which has adjoint chiral multiplets, an appropriate R -symmetry, and allows for viable breaking of $SU(5)$ by hypercharge flux.

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1 Introduction and Motivation

The last few years have seen a surge of interest in trying to construct realistic models of particle physics within F-theory [1], following foundational work in [2–7]. Much work has been done on model building and phenomenology from both a local ([8–32]) and global ([33–46]) point of view. One feature common to all studies which have appeared so far is that the ultimate aim has been to reproduce, at low scales, the physics of the minimal supersymmetric standard model (MSSM), in some appropriate corner of its parameter space. Here I wish to begin discussing the embedding in F-theory of a different class of models — those in which the gauginos acquire Dirac masses, as opposed to Majorana masses, after supersymmetry breaking. This necessarily involves extending the light spectrum, since, by definition, a Dirac fermion is two *different* left-handed spinors combined to give a single massive particle, whereas a Majorana fermion consists of only one. In order to have Dirac gauginos in a supersymmetric theory, we must therefore add to the MSSM spectrum, chiral multiplets in the adjoint representation of the gauge group. After SUSY breaking, the fermions from these multiplets can pair up with the gauginos to form Dirac gauginos.

The quantum behaviour of Dirac gauginos was studied in [47], where it was shown that they give only finite, positive radiative corrections to the squared sfermion soft masses (to be compared with the logarithmically-divergent contributions from Majorana gauginos). This means, in particular, that the gauginos can be taken to be significantly heavier than the electroweak scale, without paying a price in fine-tuning. The assumption of heavy Dirac gauginos considerably weakens the current LHC bounds on squark masses [48, 49], making this a viable alternative to so-called ‘natural SUSY’ models, in which only the third generation squarks are light enough to be produced at the LHC (see, e.g., [50–53]). Extensive work has been done on developing and studying field theory models with Dirac gauginos [54–60], and a useful overview of their properties is given in [61].

There is another good reason to be interested in Dirac gauginos: they are the only possibility in models with an unbroken (approximate) R -symmetry² at the weak scale [62–70]. This is because gauginos carry R -charge 1, and so cannot have Majorana masses in the presence of unbroken R -symmetry. R -symmetric models have the appealing property that they have far fewer soft parameters than the MSSM, and are safer from constraints on flavour and CP -violation [64]. The extra symmetry also forbids dimension four and five operators which can lead to proton decay.

We will consider the standard F-theory GUT setup: F-theory compactified on a Calabi–Yau fourfold X , elliptically-fibred over a Kähler threefold B . X will be taken to have an A_4 singularity fibred over a complex surface $S \subset B$; physically, this corresponds to having a stack of branes wrapping S , which support an $SU(5)$ gauge theory. I will refer to this

²It is not necessary to consider $U(1)_R$ invariance; the same conclusions follow from invariance under the \mathbb{Z}_p subgroup, for $p > 3$.

stack of branes as the ‘GUT brane’. $SU(5)$ will be broken to the standard model gauge group $G_{\text{SM}} \equiv SU(3) \times SU(2) \times U(1)_Y$ by turning on a non-trivial hypercharge flux on S , an approach pioneered in [5,6]. The only major difference between this work and all that which has preceded it is that the geometry and flux will be chosen so that the theory contains light chiral multiplets in the adjoint representation of G_{SM} . Since techniques to engineer a realistic matter sector have been developed at length in the references, and should translate largely unchanged to this new context, I will focus on this extended ‘adjoint sector’.

One dramatic consequence of the new chiral adjoint multiplets is that they spoil the famous unification of the gauge coupling constants in the MSSM. In four-dimensional theories, this makes it necessary to add quite a large number of extra charged fields, in incomplete $SU(5)$ multiplets, if one wishes to preserve unification [56]. In F-theory with hypercharge flux breaking of $SU(5)$, equality of all three gauge couplings at the unification scale is replaced by only a single linear condition, first written down in [71]. As this is somewhat model independent, it is discussed first, in section 2, with one notable conclusion being that in models with Dirac gauginos, F-theory unification can be achieved at around the reduced Planck scale if we also add just one vector-like pair of singlet leptons with TeV-scale mass. Section 3 then explains how to arrange for the presence of light chiral adjoints in F-theory models. After some generalities, I specialise to the case where the GUT brane wraps a complex surface $S \cong K3$, which has a number of nice features, and write down a necessary and sufficient condition for the hypercharge flux to remove all massless fields coming from the unwanted components of the adjoint of $SU(5)$. In section 4, I consider engineering a discrete R -symmetry, which must come from a geometric symmetry of the compactification. R -symmetric Dirac masses require that the adjoint superfields have R -charge zero, and the corresponding geometric condition is found explicitly, again in the $K3$ case. In section 5, a simple example is given of a compactification in which $SU(5)$ can be appropriately broken by hypercharge flux, and there is an unbroken \mathbb{Z}_4 R -symmetry under which the adjoint fields have the correct charge. Section 6 briefly concludes. There is also a short appendix reviewing some of the field-theoretic issues associated with generating Dirac gaugino masses.

2 Unification

One attractive feature of the MSSM is that the three standard model gauge couplings unify to good precision at $M_{\text{GUT}} \simeq 2.1 \times 10^{16}$ GeV. Since the new adjoint fields we wish to introduce do not fill out complete multiplets of $SU(5)$, their presence spoils this unification, forcing us to add further new charged states, although the situation in F-theory turns out to be somewhat less restrictive than in four-dimensional GUTs. Throughout this section, I will consider only one-loop running, and neglect threshold corrections, so all conclusions are somewhat qualitative.

For a gauge theory with coupling constant g , we define the fine structure constant $\alpha = \frac{g^2}{4\pi}$;

the one-loop relation between the values of this quantity measured at two energy scales μ and Λ is

$$\alpha^{-1}(\mu) = \alpha^{-1}(\Lambda) + \frac{b}{2\pi} \log\left(\frac{\mu}{\Lambda}\right) , \quad (2.1)$$

where b is a constant³ to which each charged field in the theory contributes additively.

The MSSM has three independent couplings for $U(1)_Y$, $SU(2)$, and $SU(3)$, which we can denote as g_Y, g_2, g_3 respectively. We will take the generator of $U(1)_Y$ to be embedded in $\mathfrak{su}(5)$ as $T_Y = \text{diag}(-2, -2, -2, 3, 3)$ in the fundamental representation,⁴ but to study unification, we must use an appropriately rescaled version. The generators of $SU(n)$ are conventionally taken to satisfy $\text{Tr } T^2 = \frac{1}{2}$ in the fundamental representation, so we define $T_1 = \frac{1}{\sqrt{60}} T_Y$; it is the corresponding coupling constant g_1 which we should compare to g_2 and g_3 . In the MSSM, the one-loop beta function coefficients (above all mass scales in the theory) are then

$$b_1 = -\frac{33}{5} , \quad b_2 = -1 , \quad b_3 = 3 .$$

Now consider the situation with light chiral adjoints. A chiral multiplet in the adjoint of $SU(n)$ makes a contribution $\delta b = -n$, so we find

$$\delta b_1 = 0 , \quad \delta b_2 = -2 , \quad \delta b_3 = -3 .$$

The different contributions to the three couplings mean that, if we assert their measured values at the Z pole, then they no longer unify at any scale, as can be seen in Figure 1. Note also that now $b_3 = 0$, i.e., the $SU(3)$ coupling constant no longer runs at high scales.

In F-theory, however, the presence of the chiral adjoint fields is not the only thing which interferes with standard gauge unification. Although the standard model gauge fields all arise from an underlying $SU(5)$, turning on hypercharge flux to break this to G_{SM} can already cause a discrepancy between their coupling constants at tree level [6, 71]. The details of flux breaking will be discussed in section 3, but for now it suffices to say that it involves a choice of two line bundles \mathcal{L}_a and \mathcal{L}_Y on the complex surface S on which the $SU(5)$ theory lives, and leads to the following expressions [71]:⁵

$$\begin{aligned} \alpha_1^{-1} &= \alpha_{\text{YM}}^{-1} \text{Vol}(S) - \frac{1}{2g_s} \int_S (c_1(\mathcal{L}_a)^2 + \frac{6}{5} c_1(\mathcal{L}_a) c_1(\mathcal{L}_Y) + \frac{3}{5} c_1(\mathcal{L}_Y)^2) \\ \alpha_2^{-1} &= \alpha_{\text{YM}}^{-1} \text{Vol}(S) - \frac{1}{2g_s} \int_S (c_1(\mathcal{L}_a)^2 + 2c_1(\mathcal{L}_a) c_1(\mathcal{L}_Y) + c_1(\mathcal{L}_Y)^2) \\ \alpha_3^{-1} &= \alpha_{\text{YM}}^{-1} \text{Vol}(S) - \frac{1}{2g_s} \int_S c_1(\mathcal{L}_a)^2 , \end{aligned} \quad (2.2)$$

where α_{YM} is the 8D Yang-Mills coupling, and g_s is the string coupling. The three coupling constants therefore depend (differently) on the choice of the line bundles \mathcal{L}_Y and \mathcal{L}_a , and

³Some authors use a convention in which b has the opposite sign.

⁴Traditionally, the hypercharge generator is taken to be $\frac{1}{3} T_Y$ or $\frac{1}{6} T_Y$; our normalisation is chosen so that $e^{2\pi i T_Y} = \mathbf{1}$.

⁵There are also extra terms which are sub-dominant at weak-coupling [72].

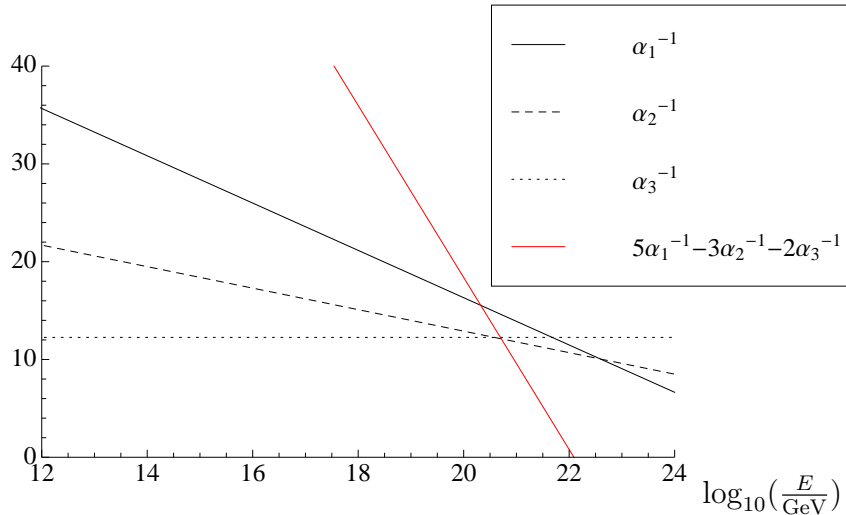


Figure 1: One-loop running of the supersymmetric standard model gauge coupling constants at high energies, in the presence of adjoint chiral multiplets. In this plot, the (Dirac) gauginos and adjoint scalars have masses of 5 TeV, while all other non-standard model states have masses of 1 TeV. We see that the relation (2.3), which defines the GUT scale, only holds well above the Planck scale.

the intersection form on S , but there is one invariant relation:

$$5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1} = 0 . \quad (2.3)$$

This is the relationship that holds between the couplings at the compactification scale.⁶ Given the low energy spectrum, we find the compactification/GUT scale by running the couplings up until (2.3) is obeyed.

If we consider just the MSSM spectrum, augmented by light chiral adjoint fields, then (2.3) is only obeyed well above the Planck scale (see Figure 1), where the calculation no longer makes sense. As such, this scenario is ruled out, and we are forced to consider the addition of further light multiplets.

2.1 Extra vector-like matter

Which extra vector-like states can we add to bring the GUT scale down to something realistic? First note that at one loop, particles contribute additively to α^{-1} ; a state of mass M changes the value of α^{-1} at scales $\mu > M$ by an amount

$$\delta\alpha^{-1} = \frac{\delta b}{2\pi} \log\left(\frac{\mu}{M}\right) .$$

To study the effect of the spectrum on the GUT scale, defined by (2.3), we must therefore consider the combination $\delta b_F := 5\delta b_1 - 3\delta b_2 - 2\delta b_3$ for each possible multiplet: the values

⁶The couplings will all be equal, as in traditional GUTs, if $\int_S (2c_1(\mathcal{L}_a)c_1(\mathcal{L}_Y) + c_1(\mathcal{L}_Y)^2) = 0$.

are given in Table 1 (the dependence of the GUT scale on extra vector-like matter, as well as threshold corrections, has also been discussed in [73, 74]).

We can see that the G_{SM} adjoints make a total contribution of $\delta b_F = 12$, which is what raises M_{GUT} from its usual value to well above the Planck scale. To bring it back down, we must introduce light states with $\delta b_F < 0$. The minimal possibility is to add one light (~ 1 TeV) vector-like pair of singlet leptons,⁷ i.e., chiral multiplets in the representation $(\mathbf{1}, \mathbf{1}, 6) \oplus (\mathbf{1}, \mathbf{1}, -6)$. The effect of this addition is to bring M_{GUT} back down to $M_{\text{GUT}} \simeq 1.7 \times 10^{18}$ GeV, which is approximately the reduced Planck scale $M_P \simeq 2.4 \times 10^{18}$ GeV. The usual small hierarchy between the Planck scale and the GUT scale is therefore removed in this scenario, resulting in a complete unification. Of course, this is not a firm prediction: if the mass and charges of the extra vector-like states are varied, then there are a number of ways to bring M_{GUT} to a reasonable value, with the only point of general concern being that all couplings remain perturbative, so that the calculations can be trusted.

$SU(5)$ irrep.	G_{SM} irrep.	δb_1	δb_2	δb_3	$\delta b_F := 5 \delta b_1 - 3 \delta b_2 - 2 \delta b_3$
10	$(\bar{\mathbf{3}}, \mathbf{1}, -4)$	$-\frac{4}{5}$	0	$-\frac{1}{2}$	-3
	$(\mathbf{3}, \mathbf{2}, 1)$	$-\frac{1}{10}$	$-\frac{3}{2}$	-1	6
	$(\mathbf{1}, \mathbf{1}, 6)$	$-\frac{3}{5}$	0	0	-3
$\bar{\mathbf{5}}$	$(\bar{\mathbf{3}}, \mathbf{1}, -2)$	$-\frac{1}{5}$	0	$-\frac{1}{2}$	0
	$(\mathbf{1}, \mathbf{2}, 3)$	$-\frac{3}{10}$	$-\frac{1}{2}$	0	0
24	$(\mathbf{8}, \mathbf{1}, 0)$	-3	0	0	6
	$(\mathbf{1}, \mathbf{3}, 0)$	0	-2	0	6
	$(\mathbf{1}, \mathbf{1}, 0)$	0	0	0	0
	$(\mathbf{3}, \mathbf{2}, -5)$	$-\frac{5}{2}$	$-\frac{3}{2}$	-1	-6
	$(\bar{\mathbf{3}}, \mathbf{2}, 5)$	$-\frac{5}{2}$	$-\frac{3}{2}$	-1	-6

Table 1: Contributions of chiral multiplets in the relevant standard model representations to the one-loop beta function coefficients. Note that a vector-like pair of any of these will make twice the given contribution.

The other interesting feature to notice in Table 1 is that the G_{SM} representations originating in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ of $SU(5)$ each have $\delta b_F = 0$, so the presence of such fields does not change M_{GUT} . We may therefore introduce any such states as messengers of SUSY breaking, or as extra Higgs doublets (as required in the ‘Minimal R -Symmetric Supersymmetric Model

⁷We will discuss in section 3 how this might be arranged in F-theory.

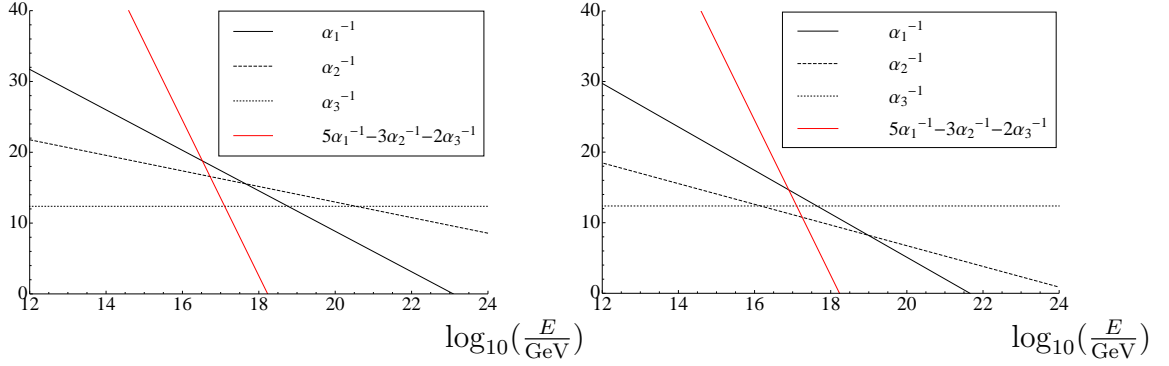


Figure 2: These plots show the running coupling constants in two different theories, each with a single light (1 TeV) vector-like pair of particles in the $(\mathbf{1}, \mathbf{1}, \pm 6)$ representation of G_{SM} . In the first, the rest of the spectrum is the same as in Figure 1, while in the second there is an extra pair of Higgs doublets, as required in the MRSSM. For the sake of the plot, these extra doublets are also given masses of 1 TeV. In each case, the couplings ‘unify’, in the F-theory sense, at $\sim 1.7 \times 10^{18}$ GeV, which is just below the reduced Planck scale.

(MRSSM) [64]), without changing M_{GUT} . Such states will have the effect of making the theory more strongly coupled at M_{GUT} , but this is not necessarily a problem. For example, even in the MRSSM with an extra vector-like pair of singlet leptons, we can add messengers filling out two full copies of $\mathbf{5} \oplus \bar{\mathbf{5}}$, with masses as low as $\sim 10^{12}$ GeV, and the largest coupling constant at M_{GUT} is $\alpha_2 \simeq \frac{1}{5}$, so the theory (just) remains perturbative.

One final point to note is the order of the couplings at M_{GUT} . The relation in (2.3) can be re-written as

$$5(\alpha_1^{-1} - \alpha_3^{-1}) = 3(\alpha_2^{-1} - \alpha_3^{-1}) ,$$

but does not say anything about the sign of these quantities; we see from (2.2) that the sign is opposite to that of $\int_S (2c_1(\mathcal{L}_a)c_1(\mathcal{L}_Y) + c_1(\mathcal{L}_Y)^2)$. As we will see in section 3, the simplest choice of flux, $c_1(\mathcal{L}_a) = 0$, makes this quantity negative, leading to the GUT-scale relation $\alpha_3^{-1} < \alpha_1^{-1} < \alpha_2^{-1}$. As shown in Figure 2, this is obeyed in the case where the adjoint chiral fields and vector-like leptons are the only addition to the MSSM spectrum, whereas the other possibility ($\alpha_2^{-1} < \alpha_1^{-1} < \alpha_3^{-1}$) occurs if we also add an extra light pair of Higgs doublets (as in the MRSSM). Note that complete $SU(5)$ multiplets do not affect such relations.

3 Hypercharge flux and massless chiral adjoints

Our starting assumption is that the complex surface S is wrapped by a stack of branes which give rise to an $SU(5)$ gauge theory in eight dimensions. As has been common in the literature since the pioneering work in [5, 6], we will then break $SU(5)$ to G_{SM} by turning on a non-trivial hypercharge flux⁸ along S . In order to preserve supersymmetry, the field

⁸It is also possible to consider discrete Wilson line breaking, as in heterotic models [75].

strength F_Y representing this flux must be of Hodge type $(1, 1)$, thus corresponding to some holomorphic line bundle \mathcal{L}_Y , and satisfy

$$F_Y \wedge \omega_S = 0, \quad (3.1)$$

where ω_S is the Kähler form on S . We must also ensure that the hypercharge gauge boson remains massless; this will be the case if $c_1(\mathcal{L}_Y) = \frac{1}{2\pi}[F_Y]$ pushes forward to zero in the cohomology of B . The dual picture is often more convenient: \mathcal{L}_Y corresponds to some divisor D_Y , which is a linear combination of algebraic curves on S , and the hypercharge gauge boson remains massless if this is homologous to zero in B .

We wish to engineer models which contain massless chiral multiplets in the adjoint representation of the gauge group, and there are two possible sources of the scalars in these multiplets: internally-polarised zero modes of the eight-dimensional gauge fields, and zero modes of the eight-dimensional scalar φ . Since the flux breaks $SU(5)$ to G_{SM} , we must decompose the adjoint representation into irreducible representations of G_{SM} , and consider each separately:

$$\begin{aligned} SU(5) &\supset SU(3) \times SU(2) \times U(1)_Y \\ \mathbf{24} &= (\mathbf{8}, \mathbf{1}, 0) \oplus (\mathbf{1}, \mathbf{3}, 0) \oplus (\mathbf{1}, \mathbf{1}, 0) \oplus (\mathbf{3}, \mathbf{2}, -5) \oplus (\bar{\mathbf{3}}, \mathbf{2}, 5), \end{aligned} \quad (3.2)$$

The gauge bundle correspondingly splits into a sum of line bundles of the form \mathcal{L}_Y^k , where k is the hypercharge of the corresponding field components. As discussed in [2, 3], the massless chiral multiplets descending from φ correspond to the cohomology groups⁹ $H^0(S, K_S \otimes \mathcal{L}_Y^k)$, while those descending from the gauge fields correspond to $H^1(S, \mathcal{L}_Y^k)$; the scalars in these multiplets are respectively the 7-brane position moduli, and continuous Wilson line moduli.

As we can see from (3.2), the fields in the adjoint of G_{SM} all have hypercharge zero, so we get massless chiral adjoints from the cohomology groups $H^0(S, K_S) \cong H^{2,0}(S)$ and $H^1(S, \mathcal{O}_S) \cong H^{0,1}(S)$. The surface S is Kähler, so its Hodge numbers satisfy $h^{p,q} = h^{q,p}$; we therefore seek surfaces with either $h^{1,0}$ or $h^{2,0}$ equal to one.

It is not clear at this stage whether we should prefer surfaces with $h^{1,0} = 1$, or $h^{2,0} = 1$. However, a particularly nice possibility is to take $S \cong K3$, the Hodge diamond for which is well known:

$$\begin{array}{ccccc} & & h^{0,0} & & 1 \\ & & & & \\ h^{1,0} & h^{0,1} & & & 0 & 0 \\ h^{2,0} & h^{1,1} & h^{0,2} & = & 1 & 20 & 1 \\ & h^{2,1} & h^{1,2} & & 0 & 0 \\ & & h^{2,2} & & & 1 \end{array}$$

⁹ K_S , K_B etc. will be used to denote both the canonical divisor (class), and the canonical line bundle. It should be clear from context which is meant.

Although other surfaces are probably just as suitable, there are several reasons to consider $S \cong K3$:

- When $h^{2,0} > 0$, the Picard number of S is generally smaller than $h^{1,1}(S)$, since the integral cohomology lattice $H^2(S, \mathbb{Z})$ need not align with the subspace of $(1, 1)$ -forms in $H^2(S, \mathbb{C})$. However, as the complex structure of S is deformed by moving it around in B , new divisor classes, not inherited from B , can appear.¹⁰ Turning on hypercharge flux corresponding to such a divisor leaves the hypercharge gauge boson massless, and also fixes some of the moduli. This mechanism is not available on a surface with $h^{2,0} = 0$.
- The trivial canonical bundle of $K3$ simplifies many calculations involving Serre duality or adjunction.
- It is very easy to find $K3$ surfaces embedded in appropriate threefolds B : any smooth anti-canonical hypersurface will be a $K3$.
- The special case $B = K3 \times \mathbb{P}^1$ is dual to the heterotic string compactified on $K3 \times T^2$. This theory has $\mathcal{N} = 2$ supersymmetry, with the adjoint chiral multiplets combining with the gauge fields to give $\mathcal{N} = 2$ vector multiplets. The actual case of interest is that in which the global geometry of B (and the flux) instead breaks this to $\mathcal{N} = 1$, but we see that $S \cong K3$ nicely realises the idea from [47] of having $\mathcal{N} = 2$ SUSY in the gauge sector only.

For these reasons we will frequently return to the case $S \cong K3$.

3.1 Absence of unwanted states

Demanding that $h^{1,0}(S) + h^{2,0}(S) = 1$ guarantees that the theory will have massless chiral multiplets filling out exactly one copy of the adjoint of G_{SM} , regardless of the hypercharge flux. We now demand that there are no light multiplets in the ‘off-diagonal’ representations appearing in the decomposition of the $SU(5)$ adjoint, (3.2). These carry five units of hypercharge, so according to the discussion in the last section, the necessary conditions are

$$H^0(S, K_S \otimes \mathcal{L}_Y^{\pm 5}) = H^1(S, \mathcal{L}_Y^{\pm 5}) = 0 .$$

As has been pointed out many times in the literature, and as we will see explicitly below, this proves impossible to satisfy in cases of interest. The solution to this problem is to consider a slightly different type of flux. We suppose that our $SU(5)$ gauge group is in fact embedded in

¹⁰This phenomenon has been used in [44] to construct G -flux in global F-theory models.

$U(5)$ (in what sense this might be true depends on global features of the compactification). The global structure of $U(5)$ is in fact

$$U(5) = \frac{SU(5) \times U(1)_a}{\mathbb{Z}_5} , \quad (3.3)$$

where $U(1)_a$ is the central ‘diagonal’ subgroup. Let T_Y be the generator of hypercharge $U(1)_Y$, and T_a be the generator of $U(1)_a$, and take the field strength of the line bundle \mathcal{L}_Y to correspond to $\frac{1}{5}(T_Y + 2T_a) = \text{diag}(0, 0, 0, 1, 1)$. Despite being a fractional linear combination, this is an appropriately normalised $U(1)$ generator, thanks to the global identification in (3.3). It is easy to see that the charges of the off-diagonal components in (3.2) with respect to this new $U(1)$ are simply ± 1 , so the conditions for the absence of exotics become $H^0(S, K_S \otimes \mathcal{L}_Y^{\pm 1}) = H^1(S, \mathcal{L}_Y^{\pm 1}) = 0$, and we will see that these are easy to satisfy.

The first thing to note is that Serre duality gives us an isomorphism

$$H^0(S, K_S \otimes \mathcal{L}_Y^{\pm 1}) = H^2(S, \mathcal{L}_Y^{\mp 1})^*,$$

so our conditions can be recast as $H^i(S, \mathcal{L}_Y^{\pm 1}) = 0$ for $i = 1, 2$. At this point it is useful to introduce the holomorphic Euler characteristic, given by $\chi(S, \mathcal{L}_Y^{\pm 1}) = \sum_{i=0}^2 (-1)^i h^i(S, \mathcal{L}_Y^{\pm 1})$. This can be calculated easily, but will only be useful if we know the value of $h^0(S, \mathcal{L}_Y^{\pm 1})$. Assuming that (3.1) holds, it is in fact easy to show that $H^0(S, \mathcal{L}_Y^{\pm 1}) = 0$. To see this, note that if \mathcal{L}_Y^k , for any $k \neq 0$, were to admit a global section, it would mean that $k c_1(\mathcal{L}_Y) = \frac{k}{2\pi} [F_Y]$ was dual to an algebraic curve $C \subset S$, and therefore

$$\int_S F_Y \wedge \omega_S = \frac{2\pi}{k} \int_C \omega_S = \frac{2\pi}{k} \text{Vol}(C) \neq 0 .$$

We conclude that $F_Y \wedge \omega_S = 0$ is sufficient to ensure that $H^0(S, \mathcal{L}_Y^k) = 0$ for all $k \neq 0$, and therefore $\chi(S, \mathcal{L}_Y^{\pm 1}) = -h^1(S, \mathcal{L}_Y^{\pm 1}) + h^2(S, \mathcal{L}_Y^{\pm 1})$.

On a complex surface S , the Hirzebruch-Riemann-Roch theorem gives the following formula for the holomorphic Euler characteristic:

$$\begin{aligned} \chi(\mathcal{L}_Y^{\pm 1}) &= \chi(\mathcal{O}_S) + \frac{1}{2}(D_Y^2 \pm D_Y \cdot K_S) \\ &= \chi(\mathcal{O}_S) + \frac{1}{2}D_Y^2 . \end{aligned}$$

The second equality here follows from our assumption that D_Y is homologically trivial in B , whereas by the adjunction formula, K_S is the restriction to S of a divisor on B , namely $S + K_B$. The first term is given in terms of the Hodge numbers of S :

$$\chi(\mathcal{O}_S) = h^{0,0}(S) - h^{1,0}(S) + h^{2,0}(S) = \begin{cases} 0 & \text{if } h^{1,0}(S) = 1, h^{2,0}(S) = 0 \\ 2 & \text{if } h^{1,0}(S) = 0, h^{2,0}(S) = 1 . \end{cases}$$

We conclude that a necessary condition to project out the unwanted states is that $D_Y^2 = 0$ in the first case, or $D_Y^2 = -4$ in the second. We see now why pure hypercharge flux cannot

work in the case $h^{2,0} = 1$: since the off-diagonal states have five units of hypercharge, the condition would be $(5D_Y)^2 = -4$, which is impossible, since D_Y is an integral class. This is what necessitates the discussion in terms of $U(5)$ which I gave above.

Interestingly, there is no such problem in the first case, as the condition remains simply $D_Y^2 = 0$. Although I will not explore this further here, it shows that pure hypercharge flux might work on surfaces with $h^{1,0} = 1$, potentially leading to some simplifications in building global models.

There does not seem to be anything more we can say in complete generality, but in the case $S \cong K3$, we can easily go further. Since $K_S \cong \mathcal{O}_S$, $H^0(S, K_S \otimes \mathcal{L}_Y^{\pm 1}) = H^0(S, \mathcal{L}_Y^{\pm 1})$, and we have already seen that the latter vanishes for a supersymmetric compactification. But now it follows from Serre duality that $H^2(S, \mathcal{L}_Y^{\pm 1}) = 0$, so in this case we have simply $\chi(\mathcal{L}_Y^{\pm 1}) = -h^1(S, \mathcal{L}_Y^{\pm 1})$, and the condition $D_Y^2 = -4$ becomes both necessary and sufficient for the vanishing of all cohomology groups.

So what are the possible choices for D_Y ? As explained above, neither D_Y nor $-D_Y$ can be effective, or (3.1) would be violated. Therefore we must write $D_Y \sim C_1 - C_2$, where each C_j is a curve on S (we assume them to be irreducible for simplicity). Then $D_Y^2 = C_1^2 + C_2^2 - 2C_1 \cdot C_2$. On a $K3$ surface, the adjunction formula gives, for any curve C ,

$$\begin{aligned} K_C &= (K_S + C)|_C = C|_C, \\ \Rightarrow \deg K_C &= \deg C|_C, \\ \Rightarrow 2g_C - 2 &= C^2, \end{aligned} \tag{3.4}$$

where g_C is the genus. Given this identity, $D_Y^2 = -4$ becomes

$$g_{C_1} + g_{C_2} - C_1 \cdot C_2 = 0.$$

It is possible to arrange for cancellation to take place here, but the simplest solution is clearly $g_{C_1} = g_{C_2} = C_1 \cdot C_2 = 0$, i.e., C_1 and C_2 are disjoint rational curves on S .¹¹ In section 5 we will write down a toy model which implements the setup we have described here.

3.2 Flux and extra vector-like states

In section 2, we discussed an appealing scenario in which there is an extra light vector-like pair of chiral multiplets in the $(\mathbf{1}, \mathbf{1}, \pm 6)$ representation of G_{SM} , leading to F-theory unification at $\sim 1.7 \times 10^{18}$ GeV. An important question to answer is whether it is possible to get such a spectrum in these models.

¹¹Taking C_1 and C_2 to be disjoint (-2) -curves will of course give $D_Y^2 = -4$ on any surface. This is therefore an appropriate choice for any surface with $h^{2,0} = 1$. The difference on $K3$ is that *any* rational curve is a (-2) -curve.

Consider the simplest situation, where the extra states arise on a particular component, C , of the **10** matter curve, with the chiral families originating on another component (the calculation of the spectrum does not always split up like this just because the matter curve is reducible; see for example [76–78]). Breaking down the $\mathbf{10} \oplus \overline{\mathbf{10}}$ under G_{SM} , the number of massless fields in each representation is given by

$$\begin{aligned} n_{(\overline{\mathbf{3}}, \mathbf{1}, -4)} &= h^0(C, \mathcal{L}') & , & \quad n_{(\mathbf{3}, \mathbf{1}, 4)} = h^1(C, \mathcal{L}') \\ n_{(\mathbf{3}, \mathbf{2}, 1)} &= h^0(C, \mathcal{L}' \otimes \mathcal{L}_Y) & , & \quad n_{(\overline{\mathbf{3}}, \mathbf{2}, -1)} = h^1(C, \mathcal{L}' \otimes \mathcal{L}_Y) \\ n_{(\mathbf{1}, \mathbf{1}, 6)} &= h^0(C, \mathcal{L}' \otimes \mathcal{L}_Y^2) & , & \quad n_{(\mathbf{1}, \mathbf{1}, -6)} = h^1(C, \mathcal{L}' \otimes \mathcal{L}_Y^2) , \end{aligned}$$

for some common line bundle \mathcal{L}' [2, 3].

For any line bundle \mathcal{L} on an algebraic curve C , we have the Riemann-Roch formula:

$$h^0(C, \mathcal{L}) - h^1(C, \mathcal{L}) = \deg(\mathcal{L}) + 1 - g_C .$$

We therefore see immediately that if we want extra $(\mathbf{1}, \mathbf{1}, \pm 6)$ states, but no others, then we must have $\deg(\mathcal{L}_Y|_C) = 0$, but $\mathcal{L}_Y|_C \neq \mathcal{O}_C$. Obviously we then require $g_C > 0$, and $\deg(\mathcal{L}'|_C) = g_C - 1$. A simple way to achieve the desired outcome is for C to be an elliptic curve, and $\mathcal{L}'|_C \cong \mathcal{L}_Y|_C \cong \mathcal{L}$, where $\mathcal{L}^{\otimes 3} = \mathcal{O}_C$, but $\mathcal{L} \neq \mathcal{O}_C$. Here we outline one example of such a geometry, without making any attempt to embed it in a consistent F-theory compactification.

First, note that if $S \cong K3$ contains a non-singular elliptic curve C , then it is in fact elliptically-fibred over \mathbb{P}^1 .¹² Let us assume that it is a special elliptic $K3$, which not only has a section, but has a non-trivial Mordell-Weil group, with torsion subgroup \mathbb{Z}_3 [79]. This means that, as well as the zero section σ , there is another section σ' such that when restricted to a generic fibre, such as C , we have $(\sigma' - \sigma)|_C \simeq 0$, but $3(\sigma' - \sigma)|_C \sim 0$. Note that σ and σ' are disjoint rational curves on S , so $\sigma' - \sigma$ can play the role of D_Y , and the desired scenario arises if the **10** matter curve contains a generic fibre C as a component, and $\mathcal{L}'|_C \cong \mathcal{L}_Y|_C \cong \mathcal{O}_C(\sigma' - \sigma)$.

The results above contradict [34], in which it was claimed that any incomplete $SU(5)$ multiplets arising on curves threaded by hypercharge flux satisfy $\delta b_F = 0$ (where again $\delta b_F = 5\delta b_1 - 3\delta b_2 - 2\delta b_3$). In that work, expressions for the δb_j are given in terms of the net chirality in each G_{SM} representation; the implicit assumption is that the vector-like fields come in complete $SU(5)$ multiplets, and so give no net contribution to δb_F . As has just been demonstrated, this is not generally true.

¹²To prove this, start with the short exact sequence

$$0 \longrightarrow \mathcal{O}_S \longrightarrow \mathcal{O}_S(C) \longrightarrow \mathcal{O}_S(C)|_C \longrightarrow 0 ,$$

and observe that the adjunction formula gives $\mathcal{O}_S(C)|_C \cong \mathcal{O}_C$, since both K_C and K_S are trivial. Taking cohomology, the above sequence then tells us that the linear system $|C|$ is one-dimensional. It cannot have any base points, since $C \cdot C = 0$, so S is elliptically-fibred over \mathbb{P}^1 .

4 R -symmetry

As mentioned earlier, perhaps the most obvious theoretical motivation for Dirac gauginos is that they are necessary in a theory with an unbroken (approximate, discrete) R -symmetry. Conversely, Dirac gauginos require some mechanism to suppress the usual Majorana mass terms, and an unbroken R -symmetry is the most obvious way to do this, at least in effective field theory.

In theories with extra dimensions, the unbroken supercharges come from covariantly-constant spinors in the compact space, and R -symmetries therefore arise from geometric symmetries which act non-trivially on these spinors. There is a slight subtlety here. The action of a geometric symmetry on tensorial quantities is always well-defined, being given by the pushforward, whereas there is a sign ambiguity in the action on spinorial quantities. However, physics does not care which sign we choose; we can see this from the fact that any Lorentz-invariant Lagrangian must contain only terms with an even number of spinors, so that an overall sign always cancels. Alternatively, note that a 2π rotation in the external dimensions is always a symmetry, and this precisely changes the sign of all spinorial quantities. These observations also show that R -parity should not really be considered an R -symmetry [80]: only symmetries of order greater than two deserve this label.

To identify possible R -symmetries in the context of F-theory, first assume that the Calabi–Yau fourfold X is smooth. Recall that if we further reduce the theory to $(2+1)$ dimensions by compactifying one space-like direction on a circle, the resulting theory is equivalent to M-theory compactified on X . The spin group in $(3+1)$ dimensions is $SL(2, \mathbb{C})$, and $\mathcal{N} = 1$ supersymmetry is generated by a doublet under this group, Q_α . Upon reduction to $(2+1)$ dimensions, the spin group becomes $SL(2, \mathbb{R}) \subset SL(2, \mathbb{C})$. The doublet of $SL(2, \mathbb{R})$ is real, so the real and imaginary parts of Q_α are now independent, and we get $\mathcal{N} = 2$ SUSY in three dimensions. From the M-theory point of view, this comes about as follows. A Calabi–Yau fourfold has holonomy group $SU(4) \subset SO(8)$, and $SO(8)$ has two eight-dimensional Majorana–Weyl spinor representations. Under $SU(4)$, one of these decomposes as $\mathbf{8} = \mathbf{6} + \mathbf{1} + \mathbf{1}$ [81, 82]; the two singlets represent the two real covariantly constant spinors, ξ_1 and ξ_2 , on X . The complex spinor $\xi = \xi_1 + i\xi_2$ then corresponds to Q_α .

In terms of ξ , the holomorphic $(4,0)$ form on X can be written as $\Omega_{Xijkl} = \xi^T \gamma_{ijkl} \xi$, where γ_{ijkl} is the anti-symmetric product of the gamma matrices with holomorphic indices. This makes it easy to search for R -symmetries: we need automorphisms $g : X \rightarrow X$ such that $g_* \Omega_X \neq \Omega_X$. For example, if $g^2 = \text{id}_X$, then we might have $g_* \Omega_X = -\Omega_X$. We must therefore have $g : \xi \rightarrow \pm i\xi$, where we are free to choose the sign, as discussed above. Such an automorphism of X would thus correspond to a \mathbb{Z}_4 R -symmetry, but of a very restricted type: since $g^2 = \text{id}_X$, all superfields carry R -charge 0 or 2. In typical R -symmetric models, the quark and lepton superfields carry R -charge 1 (such that the fermionic components are neutral), so this is not desirable. Instead, we should consider an order-four symmetry g

which satisfies $g_*\Omega_X = -\Omega_X$. This is again a \mathbb{Z}_4 R -symmetry, but now tensorial quantities can, in principle, carry any charge. The generalisation to other \mathbb{Z}_p is obvious.

In practice, of course, we are interested in singular fourfolds X , for which the M-theory dual is defined on a crepant resolution $\tilde{X} \rightarrow X$. In general, there is no reason for \tilde{X} to share the symmetries of X , but the F-theory limit is that in which all the resolution parameters (which possibly break the symmetry) vanish. So if X is the limit of some family of smooth fourfolds, all of which share a certain R -symmetry, then by continuity we expect this R -symmetry to persist in the theory defined on X , regardless of whether or not it admits a symmetric resolution. We will henceforth assume this to be true.

To detect R -symmetries we need an explicit representation of the holomorphic $(4, 0)$ -form Ω_X . To get this in some generality, assume that X is given by a smooth Weierstrass model over B . Let $P = \mathbb{P}(\mathcal{O}_B \oplus K_B^{-2} \oplus K_B^{-3})$, with homogeneous coordinates z, x, y on the fibres; then X is given by the vanishing of the Weierstrass polynomial $W = -y^2z + x^3 + fxz^2 + gz^3$, where f and g are sections of K_B^{-4} and K_B^{-6} respectively. The adjunction formula for $X \subset P$ then leads to the following short exact sequence [83]

$$0 \longrightarrow \Omega^5 P \longrightarrow \Omega^5 P(X) \xrightarrow{\text{P.R.}} \Omega^4 X \longrightarrow 0 . \quad (4.1)$$

The map labelled ‘P.R.’ here is the Poincaré residue map, given by integrating a $(5, 0)$ -form on P over the boundary of an infinitesimal tubular neighbourhood of X . We now consider the long exact sequence in cohomology following from the above.

The low-degree cohomology of the first term vanishes, which we can see as follows: B is the base of an elliptically-fibred Calabi–Yau, so $h^{p,0}(B) = 0$ for $p > 0$. Since P is a \mathbb{P}^2 bundle over B , and $h^{p,0}(\mathbb{P}^2) = 0$ for $p > 0$, this implies by the Leray spectral sequence that $h^{p,0}(P) = 0$ for $p > 0$. Since P is Kähler, we have $h^{5,q}(P) = h^{5-q,0}(P)$, and hence $H^0(P, \Omega^5 P) = H^1(P, \Omega^5 P) = 0$.

Putting the above results into the exact sequence in cohomology following from (4.1), we learn that the holomorphic $(4, 0)$ -form Ω_X is the Poincaré residue of the unique global section of $\Omega^5 P(X)$. Explicitly, if we let λ be (the pullback to P of) the unique holomorphic $(3, 0)$ -form on B with values in K_B^{-1} , then on the patch $y \neq 0$, we have

$$\Omega_X = \oint_{W=0} \frac{y \lambda \wedge dx \wedge dz}{W} . \quad (4.2)$$

The reader can check that the integrand is a well-defined meromorphic differential form on P , and that it has no extra singularities at infinity in the fibre (i.e. as $y \rightarrow 0$).

4.1 R -charge of the adjoint fields

R -symmetric Dirac masses require the adjoint chiral superfields to have zero R -charge, which occurs if their wavefunctions on S are invariant under the geometric action of the R -symmetry. This will typically need to be checked on a case-by-case basis, but we will now

see that some general statements can be made when $h^{2,0}(S) = 1$, and explicit formulae are available when $S \cong K3$.

When $h^{2,0}(S) = 1$, the unique holomorphic $(2,0)$ -form Ω_S gives rise to the massless adjoint fields.¹³ We have a short exact sequence corresponding to $S \subset B$,

$$0 \longrightarrow \Omega^3 B \longrightarrow \Omega^3 B(S) \xrightarrow{\text{P.R.}} \Omega^2 S \longrightarrow 0 .$$

Since $h^{3,0}(B) = h^{3,1}(B) = 0$ (again, because B is the base of an elliptic Calabi–Yau), and $h^{2,0}(S) = 1$ by assumption, we learn that there exists a unique meromorphic $(3,0)$ -form on B with a pole along S , and Ω_S is the Poincaré residue of this.

To go further, we need to specialise again, to the case where S is a $K3$ surface, given by the vanishing of some section $s \in \Gamma(B, K_B^{-1})$. In this case, the global section of $\Omega^3 B(S)$ can be interpreted either as a holomorphic $(3,0)$ -form with values in K_B^{-1} , or a meromorphic $(3,0)$ -form with a pole along S ; one is related to the other by dividing by the section s . This allows us to write an explicit formula for Ω_S in terms of Ω_X , as a double residue:

$$\Omega_S = \oint_{s=0} \frac{1}{s} \oint_{z=0} \frac{y \Omega_X}{x} .$$

This requires a little bit of explanation. The section $B \subset X$ is given globally by $z = 0$, but z actually has a third-order zero along B ; locally, it is x which has a simple zero along B , giving the integrand here a simple pole. We see that after the first integral, we obtain a holomorphic $(3,0)$ -form on B with values in K_B^{-1} , since Ω_X is a section of the trivial line bundle K_X , and $y/x \sim K_B^{-1}$.

We now see that in order to obtain adjoint fields with zero R -charge, we need to choose a section $s \in \Gamma(B, K_B^{-1})$ which transforms with the same charge as $y \Omega_X / x$ under the R -symmetry, so that Ω_S is invariant.

When S is not a $K3$ surface, we cannot write down a general relationship between Ω_S and Ω_X , because the global section of $\Omega^3 B(S)$ can no longer be written down in terms of Ω_X . It should still be possible to find explicit expressions in most cases, and therefore find the R -charge of the adjoint fields, but we cannot find a *general* formula.

5 A toy example

It is relatively easy to write down a geometry which realises many of the features discussed in this paper (I make no attempt at engineering a realistic matter sector).

Let $B = \mathbb{P}^3$, which is arguably the simplest threefold base we could use, with homogeneous coordinates (u_0, u_1, u_2, u_3) . Then the ambient fivefold is the projective bundle $\mathbb{P}(\mathcal{O}_B \oplus \mathcal{O}_B(8) \oplus \mathcal{O}_B(12))$, with homogeneous coordinates (z, x, y) on the fibres, in which our

¹³For $S \cong K3$, Ω_S will be everywhere non-zero; on other surfaces, it will vanish along some curve.

Calabi–Yau fourfold X is given by the vanishing of the generalised Weierstrass polynomial

$$W = -y^2z - a_1xyz - a_3yz^2 + x^3 + a_2x^2z + a_4xz^2 + a_6z^3 , \quad (5.1)$$

where a_k is a homogeneous polynomial of degree $4k$ in the u_m .

When X is smooth, we can specialise (4.2) to the case at hand to get a residue formula for the holomorphic $(4, 0)$ form:

$$\Omega_X = \oint_{W=0} \frac{y u_0 du_1 \wedge du_2 \wedge du_3 \wedge dx \wedge dz}{W} .$$

As explained in section 4, a potentially realistic R -symmetry can be obtained from an order-four automorphism $g : X \rightarrow X$ under which $\Omega_X \rightarrow -\Omega_X$. There is a simple choice here which achieves this:

$$g_4 : u_m \rightarrow i^m u_m , \quad (5.2)$$

which extends to an order-four symmetry of the fourfold X if we choose the a_k to be invariant. It is easy to check that even with this restriction, X is generically smooth.

The next step is to specify a $K3$ surface S inside $B \cong \mathbb{P}^3$, to play the role of the GUT brane. Any quartic polynomial s in \mathbb{P}^3 defines a $K3$, but a generic quartic hypersurface will have Picard number equal to one, corresponding to the hyperplane class inherited from B , and therefore we will be unable to turn on hypercharge flux. So we must choose a special family of quartics. Note that any smooth hyperplane section, C , will have self-intersection 4 in S . By the formula (3.4), C will be a curve of genus 3. One way that S can have extra divisor classes is if some of these hyperplane sections become reducible, i.e., split into a union of lower-genus curves.

In fact, it is convenient to take a slightly different point of view. In section 3, we showed that if C_1 and C_2 are disjoint rational curves in S , which are homologous in B , then turning on hypercharge flux along $D_Y = C_1 - C_2$ satisfies the criteria for a massless hypercharge gauge boson and no exotic charged states coming from the adjoint of $SU(5)$. So define two disjoint, homologous, rational curves in B :

$$C_1 = \{u_0 = u_1 = 0\} , \quad C_2 = \{u_2 = u_3 = 0\} ,$$

and now consider only those quartic hypersurfaces which contain both C_1 and C_2 .¹⁴ As explained in section 4.1, we must also choose our quartic polynomial s such that $s \rightarrow -s$ under the action of g_4 , so that the adjoint chiral superfields will have R -charge 0. Note also that the divisor class $C_1 - C_2$ is invariant under g_4 , so the hypercharge flux preserves the R -symmetry.

An explicit example, which is readily checked to be smooth, is given by

$$s = u_0 u_2^3 + u_0^2 u_3^2 + u_0^3 u_2 + u_1 u_3^3 + u_1^2 u_2^2 + u_1^3 u_3 .$$

¹⁴To make contact between the two approaches, note that S containing, say, C_1 , is equivalent to the hyperplane section $u_0 = 0$ splitting into the union of C_1 and some degree-three curve.

Finally, to specialise to those X which have an $SU(5)$ singularity along S , we must take the coefficients in (5.1) to be $a_k = s^{k-1}q_k$, where each q_k is a quartic polynomial, transforming as $q_k \rightarrow (-1)^{k+1}q_k$ under the action of g_4 , and none of them are equal to s . It is easy to check that there is enough freedom that no extra singularities necessarily occur.

The family of Calabi–Yau fourfolds constructed in this section can be used as the basis for a family of supersymmetric F-theory models with an unbroken \mathbb{Z}_4 R -symmetry, in which $SU(5)$ is broken by hypercharge flux in such a way that the $U(1)_Y$ gauge boson remains massless, and the only massless chiral fields descending from the adjoint of $SU(5)$ are those which fill out the adjoint of the standard model gauge group, which moreover have R -charge 0. This is a promising starting point for a viable SUSY model with Dirac gauginos, but there is obviously a lot more work to do to write down a complete, consistent model, and this will be deferred to future work. It may be that the particular fourfolds here are too simple for realistic model-building, but the construction illustrates the general ideas in a clear way.

6 Conclusions

In this paper I have begun the study of F-theory GUT models with Dirac gauginos. In particular, the conditions under which the requisite massless chiral adjoints arise, but ‘off-diagonal’ components of the $SU(5)$ adjoint are absent, were shown to be easily satisfied. I also showed explicitly, in the case where the visible sector resides on a $K3$ surface, how to engineer an R -symmetry under which these have the correct charge; this is potentially an important ingredient in these models, as it suppresses Majorana gaugino masses.

I have said very little about the matter sector, except to indicate how one might arrange for the presence of the light vector-like pairs required to obtain a realistic GUT scale, and nothing at all about how to break supersymmetry in a realistic way (although see appendix A for a telegraphic account of the field-theoretic considerations). These are obviously the most important next steps in developing quasi-realistic F-theory models with Dirac gauginos.

My over-arching point is the following. Given the strong bounds which the LHC has already set on MSSM-like theories, it is important to consider alternative scenarios if we wish to retain supersymmetry as a solution to the hierarchy problem. Models with Dirac gauginos are one compelling option, and it is surprising that they have basically not yet been considered by the string phenomenology community. While this work has taken only rudimentary steps, I hope that it will spark some interest in the subject.

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A Generating Dirac gaugino masses

Arranging for the low-energy theory to contain adjoint chiral multiplets is a necessary condition to have Dirac gauginos, but we must also ensure that a large Dirac mass term is generated after SUSY breaking. This appendix contains no original work, but is included to make the paper more self-contained, and to point out some of the difficulties which will need to be overcome to build realistic F-theory models with Dirac gauginos.

Denote an $SU(n)$ adjoint chiral multiplet by $\Phi = \phi + \sqrt{2}\theta\psi + \dots$, and the field strength superfield by $\mathbf{W}_\alpha = -i\lambda_\alpha + \theta_\alpha D + \dots$, with gauge indices suppressed. Dirac gaugino masses arise most simply via an interaction with a hidden sector $U(1)$ gauge field which obtains a D -term VEV. In terms of the field strength of this hidden $U(1)$, $\mathbf{W}'_\alpha = -i\lambda'_\alpha + \theta_\alpha D' + \dots$, the term we need is¹⁵

$$\mathcal{L}_D = \frac{\sqrt{2}\tilde{y}}{M} \int d^2\theta \mathbf{W}'^\alpha \text{Tr}(\mathbf{W}_\alpha \Phi) + \text{H.c.} = \frac{i\tilde{y}}{M} D' \text{Tr}(\lambda\psi) + \dots, \quad (\text{A.1})$$

where M is some mass scale, and \tilde{y} some dimensionless constant.

To generate (A.1), we can introduce a vector-like pair of chiral fields \mathbf{C}, \mathbf{C}' , in the bi-fundamental representation $(\mathbf{n}, 1)$ and its conjugate, respectively, and take their superpotential couplings to be

$$W_C = M\mathbf{C}\mathbf{C}' + y\mathbf{C}'\Phi\mathbf{C}.$$

This leads to the generation of (A.1) via the one-loop diagram shown in Figure 3. Note that this simple model is R -symmetric if we assign an R -charge of 1 to the fields \mathbf{C}, \mathbf{C}' , and 0 to the chiral adjoint superfield Φ (the latter is required for the Dirac gaugino masses to be R -symmetric).

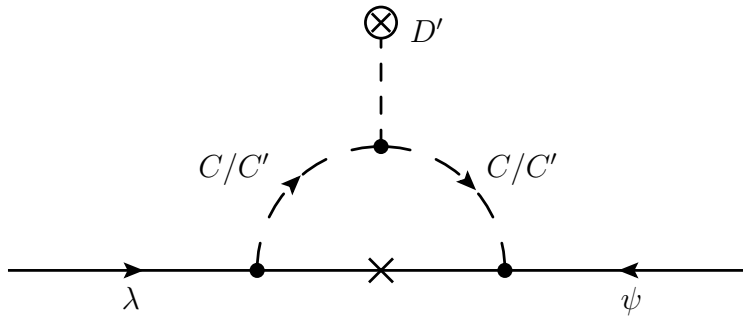


Figure 3: The one-loop diagram which generates (A.1). The crossed line represents the propagator for the Dirac fermion consisting of the superpartners of C and C' . The diagrams in which the scalar is respectively C and C' add constructively.

¹⁵Dirac masses can also be obtained from the F -term VEV of a chiral field X , via operators like $\frac{1}{M^3} \int d^4\theta X^\dagger X \text{Tr}(W^\alpha \mathcal{D}_\alpha \Phi)$, but these are typically suppressed by an extra factor of $\frac{F}{M^2}$ relative to other soft masses (although see [59] for an example where this is not the case).

Unfortunately, the simple model presented here also generates a holomorphic mass term for the adjoint scalars, leading to a tachyonic mass for one component, and thus the breaking of colour $SU(3)$, for example. This problem can be solved by taking more than one pair of messengers, and imposing certain conditions on their couplings to the adjoint fields [54], but this does pose an extra model-building challenge.

Pure D -term breaking of SUSY gives problematically-light sleptons, which led the authors of [56] to consider combined F - and D -term SUSY breaking, of the same order. This can easily be achieved in explicit models [84].

Finally, we note that the gauginos will obtain unavoidable Majorana masses from anomaly-mediation, of order $\frac{g^2}{16\pi^2}m_{3/2}$ (where $m_{3/2}$ is the gravitino mass), which we should ensure are at least an order of magnitude or two smaller than the Dirac masses, lest the benefits conferred by Dirac gauginos be lost.

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